

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

15.12 1)

$$\begin{aligned} \left(\frac{(x^2 + x) \sigma \nu \nu x}{1 - \eta \mu x} \right)' &= \frac{[(x^2 + x) \sigma \nu \nu x]' (1 - \eta \mu x) - (x^2 + x) \sigma \nu \nu x (1 - \eta \mu x)'}{(1 - \eta \mu x)^2} = \\ &= \frac{[(x^2 + x)' \sigma \nu \nu x + (x^2 + x) (\sigma \nu \nu x)'] (1 - \eta \mu x) - (x^2 + x) \sigma \nu \nu x (-\sigma \nu \nu x)}{(1 - \eta \mu x)^2} \\ &= \frac{[(2x + 1) \sigma \nu \nu x - (x^2 + x) \eta \mu x] (1 - \eta \mu x) + (x^2 + x) \sigma \nu \nu^2 x}{(1 - \eta \mu x)^2} \end{aligned}$$

15.12 2)

$$\begin{aligned} \left(\frac{x^2 (x + 1)}{x^2 + 1} \right)' &= \frac{[x^2 (x + 1)]' (x^2 + 1) - x^2 (x + 1) (x^2 + 1)'}{(x^2 + 1)^2} = \\ &= \frac{[2x (x + 1) + x^2] (x^2 + 1) - 2x^3 (x + 1)}{(x^2 + 1)^2} = \frac{(3x^2 + 2x) (x^2 + 1) - 2x^4 - 2x^3}{(x^2 + 1)^2} = \\ &= \frac{(3x^2 + 2x) (x^2 + 1) - 2x^4 - 2x^3}{(x^2 + 1)^2} = \frac{3x^4 + 3x^2 + \cancel{2x^3} + 2x - 2x^4 - \cancel{2x^3}}{(x^2 + 1)^2} = \frac{x^4 + 3x^2 + 2x}{(x^2 + 1)^2} \end{aligned}$$

15.12 3)

$$\begin{aligned} \left(\frac{x \cdot 5^x}{x + 1} \right)' &= \frac{(x \cdot 5^x)' (x + 1) - x \cdot 5^x (x + 1)'}{(x + 1)^2} = \frac{(5^x + x \cdot 5^x \ln 5) (x + 1) - x \cdot 5^x}{(x + 1)^2} = \\ &= \frac{(5^x + x \cdot 5^x \ln 5) (x + 1) - x \cdot 5^x}{(x + 1)^2} = \frac{\cancel{x \cdot 5^x} + 5^x + x^2 \cdot 5^x \ln 5 + x \cdot 5^x \ln 5 - \cancel{x \cdot 5^x}}{(x + 1)^2} = \\ &= \frac{5^x + x^2 \cdot 5^x \ln 5 + x \cdot 5^x \ln 5}{(x + 1)^2} = \end{aligned}$$

15.12 4)

$$\left(\frac{x \eta \mu x}{1 + \epsilon \phi x} \right)' = \frac{(x \eta \mu x)' (1 + \epsilon \phi x) - x \eta \mu x (1 + \epsilon \phi x)'}{(1 + \epsilon \phi x)^2} = \frac{(\eta \mu x + x \sigma \nu \nu x) (1 + \epsilon \phi x) - \frac{x \eta \mu x}{\sigma \nu \nu^2 x}}{(1 + \epsilon \phi x)^2}$$

15.12 5)

$$\left(\frac{6x^2 \ln x}{4x^2 + 3} \right)' = \frac{(6x^2 \ln x)' (4x^2 + 3) - 6x^2 \ln x (4x^2 + 3)'}{(4x^2 + 3)^2} =$$

$$= \frac{(12x \ln x + 6x)(4x^2 + 3) - 48x^3 \ln x}{(4x^2 + 3)^2} = \frac{\cancel{48x^3 \ln x} + 36x \ln x + 24x^3 + 18x - \cancel{48x^3 \ln x}}{(4x^2 + 3)^2} =$$

$$= \frac{36x \ln x + 24x^3 + 18x}{(4x^2 + 3)^2}$$

15.12 6)

$$\left(\frac{8x-5}{4x\sqrt{x}} \right)' = \frac{(8x-5)'(4x\sqrt{x}) - (8x-5)(4x\sqrt{x})'}{(4x\sqrt{x})^2} = \frac{32x\sqrt{x} - (8x-5)\left(4\sqrt{x} + \frac{2x}{\sqrt{x}}\right)}{16x^3}$$

15.12 7)

$$\left(\frac{2 \ln x}{e^x (x^2 + 5)} \right)' = \frac{(2 \ln x)' e^x (x^2 + 5) - 2 \ln x [e^x (x^2 + 5)]'}{[e^x (x^2 + 5)]^2} =$$

$$= \frac{\frac{2e^x (x^2 + 5)}{x} - 2 \ln x [e^x (x^2 + 5) + 2xe^x]}{[e^x (x^2 + 5)]^2}$$

15.12 8)

$$\left(\frac{2x\eta\mu x}{1 - \sigma\nu x} \right)' = \frac{(2x\eta\mu x)'(1 - \sigma\nu x) - 2x\eta\mu x(1 - \sigma\nu x)'}{(1 - \sigma\nu x)^2} =$$

$$= \frac{(2\eta\mu x + 2x\sigma\nu x)(1 - \sigma\nu x) - 2x\eta\mu^2 x}{(1 - \sigma\nu x)^2}$$

15.12 9)

$$\left(\frac{2x^2 + 5}{3x\eta\mu x} \right)' = \frac{(2x^2 + 5)'(3x\eta\mu x) - (2x^2 + 5)(3x\eta\mu x)'}{(3x\eta\mu x)^2} =$$

$$= \frac{12x^2\eta\mu x - (2x^2 + 5)(3\eta\mu x + 3x\sigma\nu x)}{9x^2\eta\mu^2 x}$$

15.12 10)

$$\left(\frac{2x-5}{e^x (x^2 + 4)} \right)' = \frac{(2x-5)' e^x (x^2 + 4) - (2x-5)[e^x (x^2 + 4)]'}{[e^x (x^2 + 4)]^2} =$$

$$= \frac{2e^x (x^2 + 4) - (2x-5)[e^x (x^2 + 4) + 2xe^x]}{e^{2x} (x^2 + 4)^2}$$

15.12 11)

$$\left(\frac{5\sqrt{x}}{e^x (x^2 + \sigma\varphi x)} \right)' = \frac{(5\sqrt{x})' e^x (x^2 + \sigma\varphi x) - 5\sqrt{x}[e^x (x^2 + \sigma\varphi x)]'}{[e^x (x^2 + \sigma\varphi x)]^2} =$$

$$= \frac{\frac{5e^x(x^2 + \sigma\varphi x)}{2\sqrt{x}} - 5\sqrt{x} \left[e^x(x^2 + \sigma\varphi x) + e^x \left(2x - \frac{1}{\eta\mu^2 x} \right) \right]}{e^{2x}(x^2 + \sigma\varphi x)^2}$$

15.12

12)

$$\begin{aligned} \left(\frac{x^3 3^x \eta \mu x}{x^2+7}\right)' &= \frac{\left[x^3 3^x \eta \mu x\right]'(x^2+7)-x^3 3^x \eta \mu x(x^2+7)'}{(x^2+7)^2} = \\ &= \frac{(3x^2 3^x \eta \mu x + x^3 3^x \ln 3 \eta \mu x + x^3 3^x \sigma \nu \nu x)(x^2+7)-2x^4 3^x \eta \mu x}{(x^2+7)^2} \end{aligned}$$