Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

15.11 1)

α) $$\left(\frac{1-\sigma\varphi x}{1+\sigma\varphi x}\right)'=\frac{(1-\sigma\varphi x)'(1+\sigma\varphi x)-(1-\sigma\varphi x)(1+\sigma\varphi x)'}{(1+\sigma\varphi x)^2}=$$

$$=\frac{\frac{1}{\eta\mu^2 x}(1+\sigma\varphi x)-(1-\sigma\varphi x)\frac{-1}{\eta\mu^2 x}}{(1+\sigma\varphi x)^2}=\frac{\frac{1+\cancel{\sigma\varphi x}+1-\cancel{\sigma\varphi x}}{\eta\mu^2 x}}{(1+\sigma\varphi x)^2}=\frac{2}{\eta\mu^2 x(1+\sigma\varphi x)^2}$$

β) $$\left(\frac{x-1}{x+1}-\frac{x+1}{x-1}\right)'=$$

$$=\frac{(x-1)'(x+1)-(x-1)(x+1)'}{(x+1)^2}-\frac{(x+1)'(x-1)-(x+1)(x-1)'}{(x-1)^2}$$

$$=\frac{x+1-(x-1)}{(x+1)^2}-\frac{x-1-(x+1)}{(x-1)^2}=\frac{2}{(x+1)^2}+\frac{2}{(x-1)^2}$$

15.11 2)

$$\left(\frac{x-1}{x+1}\right)'=\frac{(x-1)'(x+1)-(x-1)(x+1)'}{(x+1)^2}=\frac{1\cdot(x+1)-(x-1)\cdot 1}{(x+1)^2}=\frac{\cancel{x}+1-\cancel{x}+1}{(x+1)^2}=$$

$$=\frac{2}{(x+1)^2}$$

15.11 3)

$$\left(\frac{2x^3-1}{3x^2+2}\right)'=\frac{(2x^3-1)'(3x^2+2)-(2x^3-1)(3x^2+2)'}{(3x^2+2)^2}=\frac{6x^2(3x^2+2)-(2x^3-1)6x}{(3x^2+2)^2}=$$

$$=\frac{18x^4+12x^2-12x^4+6x}{(3x^2+2)^2}=\frac{6x^4+12x^2+6x}{(3x^2+2)^2}$$

15.11 4)

$$\left(\frac{1-\sigma\upsilon\nu x}{1+\sigma\upsilon\nu x}\right)'=\frac{(1-\sigma\upsilon\nu x)'(1+\sigma\upsilon\nu x)-(1-\sigma\upsilon\nu x)(1+\sigma\upsilon\nu x)'}{(1+\sigma\upsilon\nu x)^2}=$$

$$=\frac{\eta\mu x(1+\sigma\upsilon\nu x)-(1-\sigma\upsilon\nu x)(-\eta\mu x)}{(1+\sigma\upsilon\nu x)^2}=\frac{\eta\mu x+\cancel{\eta\mu x\sigma\upsilon\nu x}+\eta\mu x-\cancel{\eta\mu x\sigma\upsilon\nu x}}{(1+\sigma\upsilon\nu x)^2}=$$

$$=\frac{2\eta\mu x}{(1+\sigma\upsilon\nu x)^2}$$

15.11 5)

$$\left(\frac{1-\varepsilon\varphi x}{1+\varepsilon\varphi x}\right)'=\frac{(1-\varepsilon\varphi x)'(1+\varepsilon\varphi x)-(1-\varepsilon\varphi x)(1+\varepsilon\varphi x)'}{(1+\varepsilon\varphi x)^2}=$$

$$=\frac{\frac{-1-\varepsilon\varphi x}{\sigma\upsilon\nu^2 x}-\frac{1-\varepsilon\varphi x}{\sigma\upsilon\nu^2 x}}{(1+\varepsilon\varphi x)^2}=\frac{\frac{-1-\varepsilon\varphi x}{\sigma\upsilon\nu^2 x}-\frac{1-\varepsilon\varphi x}{\sigma\upsilon\nu^2 x}}{(1+\varepsilon\varphi x)^2}=\frac{\frac{-1-\cancel{\varepsilon\varphi x}-1+\cancel{\varepsilon\varphi x}}{\sigma\upsilon\nu^2 x}}{(1+\varepsilon\varphi x)^2}=\frac{-2}{\sigma\upsilon\nu^2 x(1+\varepsilon\varphi x)^2}$$

15.11 6)

$$\left(\frac{x-\ln x}{x^2+1}\right)'=\frac{(x-\ln x)'(x^2+1)-(x-\ln x)(x^2+1)'}{(x^2+1)^2}=$$

$$\frac{\left(1-\frac{1}{x}\right)(x^2+1)-(x-\ln x)2x}{(x^2+1)^2}$$

15.11 7)

$$\left(\frac{e^x-1}{e^x+1}\right)'=\frac{(e^x-1)'(e^x+1)-(e^x-1)(e^x+1)'}{(e^x+1)^2}=\frac{e^x(e^x+1)-(e^x-1)e^x}{(e^x+1)^2}=$$

$$=\frac{e^x\left[(e^x+1)-(e^x-1)\right]}{(e^x+1)^2}=\frac{e^x\left(\cancel{e^x}+1-\cancel{e^x}+1\right)}{(e^x+1)^2}=\frac{2e^x}{(e^x+1)^2}$$

15.11 8)

$$\left(\frac{x^2-\eta\mu x}{x+\sigma\varphi x}\right)'=\frac{(x^2-\eta\mu x)'(x+\sigma\varphi x)-(x^2-\eta\mu x)(x+\sigma\varphi x)'}{(x+\sigma\varphi x)^2}=$$

$$=\frac{(2x-\sigma\upsilon\nu x)(x+\sigma\varphi x)-(x^2-\eta\mu x)\left(1-\frac{1}{\eta\mu^2 x}\right)}{(x+\sigma\varphi x)^2}$$

15.11 9)

$$\left(\frac{3^x-5\sqrt{x}}{x^2+x-1}\right)'=\frac{(3^x-5\sqrt{x})'(x^2+x-1)-(3^x-5\sqrt{x})(x^2+x-1)'}{(x^2+x-1)^2}=$$

$$=\frac{\left(3^x\ln 2-\frac{5}{2\sqrt{x}}\right)(x^2+x-1)-(3^x-5\sqrt{x})(2x+1)}{(x^2+x-1)^2}=$$

15.11 10)

$$\left(\frac{\sigma\varphi x-x}{2^x-5}\right)'=\frac{(\sigma\varphi x-x)'(2^x-5)-(\sigma\varphi x-x)(2^x-5)'}{(2^x-5)^2}=$$

$$=\frac{\left(-\frac{1}{\eta\mu^2 x}-1\right)(2^x-5)-(\sigma\varphi x-x)2^x\ln 2}{(2^x-5)^2}=$$

15.11 11)

$$\left(\frac{2\eta\mu x}{1+\sigma\upsilon\nu x}\right)'=\frac{(2\eta\mu x)'(1+\sigma\upsilon\nu x)-(2\eta\mu x)(1+\sigma\upsilon\nu x)'}{(1+\sigma\upsilon\nu x)^2}=$$

$$=\frac{(2\sigma\upsilon\nu x)(1+\sigma\upsilon\nu x)-(2\eta\mu x)(-\eta\mu x)}{(1+\sigma\upsilon\nu x)^2}=\frac{2\sigma\upsilon\nu x+2\sigma\upsilon\nu^2 x+2\eta\mu^2 x}{(1+\sigma\upsilon\nu x)^2}=$$

$$=\frac{2\sigma\upsilon\nu x+2\left(\sigma\upsilon\nu^2x+\eta\mu^2x\right)}{\left(1+\sigma\upsilon\nu x\right)^2}\overset{\sigma\upsilon\nu^2x+\eta\mu^2x=1}{=}\frac{2\sigma\upsilon\nu x+2}{\left(1+\sigma\upsilon\nu x\right)^2}$$

15.11 12)

$$\left(\frac{\eta\mu x}{x}-\frac{\sigma\upsilon\nu x}{e^x}\right)'=\frac{\left(\eta\mu x\right)'x-\eta\mu x\left(x\right)'}{x^2}-\frac{\left(\sigma\upsilon\nu x\right)'e^x-\sigma\upsilon\nu x\left(e^x\right)'}{\left(e^x\right)^2}=$$

$$=\frac{x\sigma\upsilon\nu x-\eta\mu x}{x^2}-\frac{-\eta\mu x e^x-e^x\sigma\upsilon\nu x}{e^{2x}}=\frac{x\sigma\upsilon\nu x-\eta\mu x}{x^2}+\frac{\eta\mu x e^x+e^x\sigma\upsilon\nu x}{e^{2x}}$$

15.11 13)

$$\left(\frac{x-6}{x+6}+\frac{x+6}{x-6}\right)'=$$

$$=\frac{x+6-\left(x-6\right)}{\left(x+6\right)^2}+\frac{x-6-\left(x+6\right)}{\left(x-6\right)^2}=\frac{\cancel{x}+6-\cancel{x}+6}{\left(x+6\right)^2}+\frac{\cancel{x}-6-\cancel{x}-6}{\left(x-6\right)^2}=$$

$$=\frac{12}{\left(x+6\right)^2}-\frac{12}{\left(x-6\right)^2}=$$

15.11 14)

$$\left(\frac{x^2-16}{x^2+16}-\frac{x^2+16}{x^2-16}\right)'=$$

$$=\frac{\left(x^2-16\right)'\left(x^2+16\right)-\left(x^2-16\right)\left(x^2+16\right)'}{\left(x^2+16\right)^2}-\frac{\left(x^2+16\right)'\left(x^2-16\right)-\left(x^2+16\right)\left(x^2-16\right)'}{\left(x^2-16\right)^2}=$$

$$=\frac{2x\left(x^2+16\right)-\left(x^2-16\right)2x}{\left(x^2+16\right)^2}-\frac{2x\left(x^2-16\right)-\left(x^2+16\right)2x}{\left(x^2-16\right)^2}=$$

$$=\frac{2x\left[\left(x^2+16\right)-\left(x^2-16\right)\right]}{\left(x^2+16\right)^2}-\frac{2x\left[\left(x^2-16\right)-\left(x^2+16\right)\right]}{\left(x^2-16\right)^2}=$$

$$=\frac{2x\left(\cancel{x^2}+16-\cancel{x^2}+16\right)}{\left(x^2+16\right)^2}-\frac{2x\left(\cancel{x^2}-16-\cancel{x^2}-16\right)}{\left(x^2-16\right)^2}=$$

$$=\frac{64x}{\left(x^2+16\right)^2}-\frac{-64x}{\left(x^2-16\right)^2}=\frac{64x}{\left(x^2+16\right)^2}+\frac{64x}{\left(x^2-16\right)^2}$$