

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

14.8 1)

$$\begin{aligned} \boxed{g'(3)} &= \lim_{x \rightarrow 3} \frac{g(x) - g(3)}{x - 3} \stackrel{g(x) = (x^2 - 4x + 3)f(x) \quad g(3) = (3^2 - 4 \cdot 3 + 3)f(3) = 0}{=} \lim_{x \rightarrow 3} \frac{(x^2 - 4x + 3)f(x)}{x - 3} \stackrel{(x^2 - 4x + 3) = (x-3)(x-1)}{=} \\ &= \lim_{x \rightarrow 3} \frac{\cancel{(x-3)}(x-1)f(x)}{\cancel{x-3}} \stackrel{f: \text{συνεχής στο } x_0=3 \Rightarrow \lim_{x \rightarrow 3} f(x) = f(3)}{=} (3-1)f(3) = \boxed{2f(3)} \end{aligned}$$

14.8 2)

$$\begin{aligned} \boxed{g'(2)} &= \lim_{x \rightarrow 2} \frac{g(x) - g(2)}{x - 2} \stackrel{g(x) = (x^2 - x - 2)f(x) \quad g(2) = (2^2 - 2 - 2)f(2) = 0}{=} \lim_{x \rightarrow 2} \frac{(x^2 - x - 2)f(x)}{x - 2} \stackrel{(x^2 - x - 2) = (x-2)(x+1)}{=} \\ &= \lim_{x \rightarrow 2} \frac{\cancel{(x-2)}(x+1)f(x)}{\cancel{x-2}} \stackrel{f: \text{συνεχής στο } x_0=2 \Rightarrow \lim_{x \rightarrow 2} f(x) = f(2)}{=} (2+1)f(2) = \boxed{3f(2)} \end{aligned}$$

14.8 3)

$$\begin{aligned} \boxed{g'(-1)} &= \lim_{x \rightarrow -1} \frac{g(x) - g(-1)}{x + 1} \stackrel{g(x) = (x^2 - 3x - 4)f(x) = (x-4)(x+1)f(x) \quad g(-1) = [(-1)^2 - 3(-1) - 4]f(-1) = 0}{=} \\ &= \lim_{x \rightarrow -1} \frac{(x-4)\cancel{(x+1)}f(x)}{\cancel{x+1}} \stackrel{f: \text{συνεχής στο } x_0=-1 \Rightarrow \lim_{x \rightarrow -1} f(x) = f(-1)}{=} (-1-4)f(-1) = \boxed{-5f(-1)} \end{aligned}$$