Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

14.5 1)

$$\boxed{f'(1)}=\lim_{x\to1}\frac{f(x)-f(1)}{x-1}\overset{\substack{f(x)=\sqrt{x+3}\\ f(1)=2}}{=}\lim_{x\to1}\frac{\sqrt{x+3}-2}{x-1}=\lim_{x\to1}\frac{\left(\sqrt{x+3}-2\right)\left(\sqrt{x+3}+2\right)}{(x-1)\left(\sqrt{x+3}+2\right)}=$$

$$=\lim_{x\to1}\frac{\sqrt{x+3}^2-4}{(x-1)\left(\sqrt{x+3}+2\right)}=\lim_{x\to1}\frac{\cancel{x-1}}{\cancel{(x-1)}\left(\sqrt{x+3}+2\right)}=\frac{1}{\sqrt{1+3}+2}=\boxed{\frac{1}{4}}$$

14.5 2)

$$\boxed{f'(6)}=\lim_{x\to6}\frac{f(x)-f(6)}{x-6}\overset{\substack{f(x)=\sqrt{3x-2}\\ f(6)=4}}{=}\lim_{x\to6}\frac{\sqrt{3x-2}-4}{x-6}=$$

$$=\lim_{x\to6}\frac{\left(\sqrt{3x-2}-4\right)\left(\sqrt{3x-2}+4\right)}{(x-6)\left(\sqrt{3x-2}+4\right)}=\lim_{x\to6}\frac{\sqrt{3x-2}^2-4^2}{(x-6)\left(\sqrt{3x-2}+4\right)}=$$

$$=\lim_{x\to6}\frac{3x-18}{(x-6)\left(\sqrt{3x-2}+4\right)}=\lim_{x\to6}\frac{3\cancel{(x-6)}}{\cancel{(x-6)}\left(\sqrt{3x-2}+4\right)}=\frac{3}{\sqrt{3\cdot6-2}+4}=\boxed{\frac{3}{8}}$$

14.5 3)

$$\boxed{f'(-2)}=\lim_{x\to-2}\frac{f(x)-f(-2)}{x+2}\overset{\substack{f(x)=\sqrt{x^2+3x+6}\\ f(-2)=2}}{=}\lim_{x\to-2}\frac{\sqrt{x^2+3x+6}-2}{x+2}=$$

$$=\lim_{x\to-2}\frac{\left(\sqrt{x^2+3x+6}-2\right)\left(\sqrt{x^2+3x+6}+2\right)}{(x+2)\left(\sqrt{x^2+3x+6}+2\right)}=\lim_{x\to-2}\frac{\sqrt{x^2+3x+6}^2-2^2}{(x+2)\left(\sqrt{x^2+3x+6}+2\right)}=$$

$$=\lim_{x\to-2}\frac{x^2+3x+6-4}{(x+2)\left(\sqrt{x^2+3x+6}+2\right)}=\lim_{x\to-2}\frac{x^2+3x+2}{(x+2)\left(\sqrt{x^2+3x+6}+2\right)}=$$

$$\overset{x^2+3x+2=(x+1)(x+2)}{=}\lim_{x\to-2}\frac{(x+1)\cancel{(x+2)}}{\cancel{(x+2)}\left(\sqrt{x^2+3x+6}+2\right)}=\frac{-2+1}{\sqrt{(-2)^2+3(-2)+6}+2}=\boxed{-\frac{1}{4}}$$

14.5 4)

$$f'(-1)=\lim_{h\to0}\frac{f(-1+h)-f(-1)}{h}\overset{\substack{f(-1+h)=\sqrt{(-1+h)^2-3(-1+h)}\\ f(-1)=2}}{=}$$

$$=\lim_{h\to0}\frac{\sqrt{1-2h+h^2+3-3h}-2}{h}=\lim_{h\to0}\frac{\sqrt{h^2-5h+4}-2}{h}=$$

$$=\lim_{h\to0}\frac{\sqrt{h^2-5h+4}^2-2^2}{h\left(\sqrt{h^2-5h+4}+2\right)}=\lim_{h\to0}\frac{h^2-5h+\cancel{4}-\cancel{4}}{h\left(\sqrt{h^2-5h+4}+2\right)}=$$

$$=\lim_{h\to0}\frac{\cancel{h}(h-5)}{\cancel{h}\left(\sqrt{h^2-5h+4}+2\right)}=\frac{0-5}{\sqrt{0^2-5\cdot0+4}+2}=-\frac{5}{4}$$