Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

14.4 1)

α) $$f'(-1)=\lim_{x\to-1}\frac{f(x)-f(-1)}{x+1}\overset{\substack{f(x)=\frac{4}{x-1}\\ f(-1)=-2}}{=}\lim_{x\to-1}\frac{\frac{4}{x-1}+2}{x+1}=\lim_{x\to-1}\frac{\frac{4+2(x-1)}{x-1}}{x+1}$$

$$=\lim_{x\to-1}\frac{4+2x-2}{(x+1)(x-1)}=\lim_{x\to-1}\frac{2x+2}{(x+1)(x-1)}=\lim_{x\to-1}\frac{2\cancel{(x+1)}}{\cancel{(x+1)}(x-1)}=\frac{2}{-1-1}=-1$$

β) $$f'(3)=\lim_{h\to0}\frac{f(3+h)-f(3)}{h}\overset{f(3+h)=\frac{(3+h)+1}{(3+h)-2}}{=}\lim_{h\to0}\frac{\frac{(3+h)+1}{(3+h)-2}-\frac{3+1}{3-2}}{h}=$$

$$=\lim_{h\to0}\frac{\frac{h+4}{h+1}-4}{h}=\lim_{h\to0}\frac{\frac{h\cancel{+4}-4h\cancel{-4}}{h+1}}{h}=\lim_{h\to0}\frac{-3\cancel{h}}{\cancel{h}(h+1)}=\frac{-3}{0+1}=-3$$

14.4 2)

$$\boxed{f'(-2)}=\lim_{h\to0}\frac{f(-2+h)-f(-2)}{h}\overset{f(-2+h)=\frac{3}{(-2+h)+1}}{=}\lim_{h\to0}\frac{\frac{3}{(-2+h)+1}-\frac{3}{-2+1}}{h}=$$

$$=\lim_{h\to0}\frac{\frac{3}{h-1}+3}{h}=\lim_{h\to0}\frac{\frac{\cancel{3}+3h-\cancel{3}}{h-1}}{h}=\lim_{h\to0}\frac{3\cancel{h}}{\cancel{h}(h-1)}=\frac{3}{-1}=\boxed{-3}$$

14.4 3)

$$\boxed{f'\left(-\frac{1}{3}\right)}=\lim_{h\to0}\frac{f\left(-\frac{1}{3}+h\right)-f\left(-\frac{1}{3}\right)}{h}\overset{\substack{f\left(-\frac{1}{3}+h\right)=\frac{2}{-\frac{1}{3}+h}\\ f\left(-\frac{1}{3}\right)=\frac{2}{-\frac{1}{3}}=-6}}{=}\lim_{h\to0}\frac{\frac{2}{-\frac{1}{3}+h}+6}{h}=\lim_{h\to0}\frac{\frac{6}{-1+3h}+6}{h}=$$

$$=\lim_{h\to0}\frac{\frac{\cancel{6}-\cancel{6}+18h}{-1+3h}}{h}=\lim_{h\to0}\frac{18\cancel{h}}{\cancel{h}(-1+3h)}=\boxed{-18}$$

14.4 4)

$$\boxed{f'(-1)}=\lim_{h\to0}\frac{f(-1+h)-f(-1)}{h}\overset{\substack{f(-1+h)=\frac{-1+h}{-1+h+2}\\ f(-1)=-1}}{=}\lim_{h\to0}\frac{\frac{-1+h}{-1+h+2}+1}{h}$$

$$=\lim_{h\to0}\frac{\frac{-\cancel{1}+h+h+\cancel{1}}{h+1}}{h}=\lim_{h\to0}\frac{2\cancel{h}}{\cancel{h}(h+1)}=\boxed{2}$$

14.4 5)

$$\boxed{f'(0)}=\lim_{x\to0}\frac{f(x)-f(0)}{x-0}\overset{\substack{f(x)=\frac{x^2}{x^2+1}\\ f(0)=0}}{=}\lim_{x\to0}\frac{\frac{x^{\cancel{2}}}{x^2+1}}{\cancel{x}}=\lim_{x\to0}\frac{x}{x^2+1}=\frac{0}{0^2+1}=\boxed{0}$$