

α) Θέτουμε $h(x) = \frac{f(x)}{\sqrt{x}-1}$, οπότε $\lim_{x \rightarrow 1} h(x) = 12$ και ακόμη

$$h(x) = \frac{f(x)}{\sqrt{x}-1} \Rightarrow f(x) = (\sqrt{x}-1)h(x) \Rightarrow$$

$$\Rightarrow \lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} (\sqrt{x}-1)h(x) \stackrel{\lim_{x \rightarrow 1} h(x)=12}{\Rightarrow} \lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} (\sqrt{1}-1) \cdot 12$$

$$\Rightarrow \lim_{x \rightarrow 1} f(x) = 0 \stackrel{\substack{f: \text{συνεχής στο } x_0=1 \\ (\text{διότι είναι παραγωγίσιμη})}}{\Rightarrow} \boxed{f(1) = 0}$$

β) $\boxed{f'(1)} = \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} \stackrel{f(1)=0}{=} \lim_{x \rightarrow 1} \frac{f(x)}{x - 1} \stackrel{f(x)=(\sqrt{x}-1)h(x)}{=} \lim_{x \rightarrow 1} \frac{(\sqrt{x}-1)h(x)}{x - 1} =$

$$= \lim_{x \rightarrow 1} \frac{(\sqrt{x}-1)h(x)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)} = \lim_{x \rightarrow 1} \frac{(\sqrt{x}^2-1)h(x)}{(x-1)(\sqrt{x}+1)} =$$

$$= \lim_{x \rightarrow 1} \frac{\cancel{(x-1)}h(x)}{\cancel{(x-1)}(\sqrt{x}+1)} \stackrel{\lim_{x \rightarrow 1} h(x)=12}{=} \frac{12}{\sqrt{1}+1} = \boxed{6}$$

γ) Θέτουμε $\varphi(x) = \frac{g(x)}{x-1}$, οπότε $\lim_{x \rightarrow 1} \varphi(x) = 3$ και ακόμη

$$\varphi(x) = \frac{g(x)}{x-1} \Rightarrow g(x) = (x-1)\varphi(x) \Rightarrow \lim_{x \rightarrow 1} g(x) = \lim_{x \rightarrow 1} (x-1)\varphi(x) \Rightarrow$$

$$\stackrel{\lim_{x \rightarrow 1} \varphi(x)=3}{\Rightarrow} \lim_{x \rightarrow 1} g(x) = (1-1) \cdot 3 \Rightarrow \lim_{x \rightarrow 1} g(x) = 0 \stackrel{\substack{g: \text{συνεχής στο } x_0=1 \\ (\text{διότι είναι παραγωγίσιμη})}}{\Rightarrow} \boxed{g(1) = 0}$$

Ακόμη

$$\boxed{g'(1)} = \lim_{x \rightarrow 1} \frac{g(x) - g(1)}{x - 1} \stackrel{g(1)=0}{=} \lim_{x \rightarrow 1} \frac{g(x)}{x - 1} = \boxed{3}$$

δ) $\lim_{x \rightarrow 1} \frac{f(x)}{g(x)} \stackrel{f(1)=g(1)=0}{=} \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{g(x) - g(1)} \stackrel{\text{διατρούμε με } x-1}{=} \lim_{x \rightarrow 1} \frac{\frac{f(x) - f(1)}{x - 1}}{\frac{g(x) - g(1)}{x - 1}} =$

$$= \frac{\lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1}}{\lim_{x \rightarrow 1} \frac{g(x) - g(1)}{x - 1}} = \frac{f'(1)}{g'(1)} = \frac{6}{3} = \boxed{2}$$