Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

14.2 1)

$$\boxed{f'(2)}=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}\overset{\substack{f(2+h)=5(2+h)^2-(2+h)+1\\ f(2)=19}}{=}\lim_{h\to 0}\frac{5(2+h)^2-(2+h)+1-19}{h}=$$

$$=\lim_{h\to 0}\frac{5(4+4h+h^2)-2-h-18}{h}=\lim_{h\to 0}\frac{\cancel{20}+20h+5h^2-h-\cancel{20}}{h}$$

$$=\lim_{h\to 0}\frac{19h+5h^2}{h}=\lim_{h\to 0}\frac{\cancel{h}(19+5h)}{\cancel{h}}=\boxed{19}$$

14.2 2)

$$\boxed{f'(1)}=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}\overset{\substack{f(1+h)=3(1+h)-2\\ f(1)=1}}{=}\lim_{h\to 0}\frac{3(1+h)-2-1}{h}=$$

$$=\lim_{h\to 0}\frac{\cancel{3}+3h-\cancel{3}}{h}=\lim_{h\to 0}\frac{3\cancel{h}}{\cancel{h}}=\boxed{3}$$

14.2 3)

$$\boxed{f'(4)}=\lim_{h\to 0}\frac{f(4+h)-f(4)}{h}\overset{\substack{f(4+h)=-2(4+h)-5\\ f(4)=-13}}{=}\lim_{h\to 0}\frac{-2(4+h)-5+13}{h}=$$

$$=\lim_{h\to 0}\frac{-\cancel{8}-2h+\cancel{8}}{h}=\lim_{h\to 0}\frac{-2\cancel{h}}{\cancel{h}}=\boxed{-2}$$

14.2 4)

$$\boxed{f'(4)}=\lim_{h\to 0}\frac{f(4+h)-f(4)}{h}\overset{\substack{f(4+h)=(4+h)^2\\ f(4)=16}}{=}\lim_{h\to 0}\frac{(4+h)^2-16}{h}=\lim_{h\to 0}\frac{\cancel{16}+8h+h^2-\cancel{16}}{h}=$$

$$=\lim_{h\to 0}\frac{8h+h^2}{h}=\lim_{h\to 0}\frac{\cancel{h}(8+h)}{\cancel{h}}=8+0=8$$

14.2 5)

$$\boxed{f'(-5)}=\lim_{h\to 0}\frac{f(-5+h)-f(-5)}{h}\overset{\substack{f(-5+h)=-2(-5+h)^2\\ f(-5)=-50}}{=}\lim_{h\to 0}\frac{-2(-5+h)^2-(-50)}{h}=$$

$$=\lim_{h\to 0}\frac{-2(25-10h+h^2)+50}{h}=\lim_{h\to 0}\frac{\cancel{-50}+20h-2h^2+\cancel{50}}{h}=\lim_{h\to 0}\frac{\cancel{h}(20-2h)}{\cancel{h}}=$$

$$=20-2\cdot 0=20$$

14.2 6)

$$\boxed{f'(-5)}=\lim_{h\to 0}\frac{f(-5+h)-f(-5)}{h}\overset{\substack{f(-5+h)=(-5+h)^2+3\\ f(-5)=28}}{=}\lim_{h\to 0}\frac{(-5+h)^2+3-28}{h}=$$

$$=\lim_{h\to 0}\frac{h^2-10h+\cancel{25}-\cancel{25}}{h}=\lim_{h\to 0}\frac{\cancel{h}(h-10)}{\cancel{h}}=0-10=\boxed{-10}$$

14.2 7)

$$\boxed{f'(2)} = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h} \overset{\substack{f(2+h)=2(2+h)^2-(2+h)\\ f(2)=6}}{=} \lim_{h\to 0}\frac{2(2+h)^2-(2+h)-6}{h} =$$

$$= \lim_{h\to 0}\frac{2(4+4h+h^2)-2-h-6}{h} = \lim_{h\to 0}\frac{\cancel{8}+8h+2h^2-h-\cancel{8}}{h}$$

$$= \lim_{h\to 0}\frac{7h+2h^2}{h} = \lim_{h\to 0}\frac{\cancel{h}(7+2h)}{\cancel{h}} = \boxed{7}$$

14.2 8)

$$\boxed{f'(1)} = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h} \overset{\substack{f(1+h)=3(1+h)^2+(1+h)-2\\ f(1)=2}}{=} \lim_{h\to 0}\frac{3(1+h)^2+(1+h)-2-2}{h} =$$

$$= \lim_{h\to 0}\frac{3(1+2h+h^2)+1+h-2-2}{h} = \lim_{h\to 0}\frac{\cancel{3}+6h+3h^2+\cancel{1}+h-\cancel{2}-\cancel{2}}{h} =$$

$$= \lim_{h\to 0}\frac{3h^2+7h}{h} = \lim_{h\to 0}\frac{\cancel{h}(3h+7)}{\cancel{h}} = 3\cdot 0+7 = 7$$