1.6 1)

α) $f(x)=f(0)\Rightarrow x^2-2x+3=0^2-2\cdot 0+3\Rightarrow x^2-2x+3=3\Rightarrow x^2-2x=0\Rightarrow$

$\Rightarrow x(x-2)=0\Rightarrow \boxed{x=0 \text{ ή } x=2}$

β) $f(x)-f(-1)=0\Rightarrow \left(x^2-2x+3\right)-\left[(-1)^2-2(-1)+3\right]=0\Rightarrow$

$x^2-2x+3-6=0\Rightarrow x^2-2x-3=0$

$\Delta=4+12=16 \quad x_{1,2}=\dfrac{2\pm 4}{2}\Rightarrow \boxed{\begin{array}{l} x_1=-1 \\ \\ x_2=3 \end{array}}$

1.6 2)

$f(2x-1)+f(x+2)=-2\Rightarrow 3(2x-1)+2+3(x+2)+2=-2\Rightarrow$

$\Rightarrow 6x-3+2+3x+6+2=-2\Rightarrow 9x=-9\Rightarrow \boxed{x=-1}$

1.6 3)

$5-2f(2x-3)=f(1)+f(x-2)\Rightarrow 5-2\left[7(2x-3)-4\right]=7\cdot 1-4+7(x-2)-4\Rightarrow$

$\Rightarrow 5-2(14x-21-4)=7-4+7x-14-4\Rightarrow 5-28x+42+8=7x-15\Rightarrow$

$\Rightarrow 5-28x+42+8=7x-15\Rightarrow -28x+55=7x-15\Rightarrow -35x=-70\Rightarrow \boxed{x=2}$

1.6 4)

Με περιορισμούς $x\neq 1$, $x\neq 3$ και $\dfrac{x+2}{x-3}\neq 1$ έχουμε

$$f\left(\frac{x+2}{x-3}\right)=3x^2 \overset{f(x)=\frac{3x+2}{x-1}}{\Rightarrow} \frac{3\dfrac{x+2}{x-3}+2}{\dfrac{x+2}{x-3}-1}=3x^2\Rightarrow \frac{\dfrac{3x+\cancel{6}+2x-\cancel{6}}{x-3}}{\dfrac{\cancel{x}+2-\cancel{x}+3}{x-3}}=3x^2\Rightarrow \frac{\dfrac{5x}{\cancel{x-3}}}{\dfrac{5}{\cancel{x-3}}}=3x^2\Rightarrow$$

$\Rightarrow x=3x^2\Rightarrow 3x^2-x=0\Rightarrow x(3x-1)=0\Rightarrow x=0 \text{ ή } x=\dfrac{1}{3}$