

$$\left. \begin{array}{l} f(f(x+y)) = x - f(y) \xRightarrow{\text{θέτουμε } y=0} f(f(x)) = x - f(0) \\ \text{και} \\ f(f(x+y)) = x - f(y) \xRightarrow{\text{θέτουμε } x=0, y=x} f(f(x)) = 0 - f(x) \end{array} \right\} \begin{array}{l} \text{πρώτα μέλη ίσα} \\ \Rightarrow -f(x) = x - f(0) \Rightarrow \end{array}$$

$$\Rightarrow f(x) = -x + f(0) \quad (1)$$

Απομένει, να αποδειχτεί ότι $f(0) = 0$. Πράγματι

$$\left. \begin{array}{l} f(f(x+y)) = x - f(y) \xRightarrow{\text{θέτουμε } x=y=0} f(f(0)) = -f(0) \quad (1) \\ \text{και} \\ (1) \xRightarrow{\text{θέτουμε } x=f(0)} f(f(0)) = -f(0) + f(0) \Rightarrow f(f(0)) = 0 \end{array} \right\} \begin{array}{l} \text{πρώτα μέλη ίσα} \\ \Rightarrow -f(0) = 0 \Rightarrow f(0) = 0 \quad (2) \end{array}$$

$$\text{Οπότε } (1), (2) \Rightarrow \boxed{f(x) = -x}$$