

$$\alpha) \quad f(x) \cdot f(y) - f(xy) = x + y \quad \xRightarrow{\text{θέτουμε } x=y=1} f(1) \cdot f(1) - f(1 \cdot 1) = 1 + 1 \Rightarrow$$

$$f^2(1) - f(1) - 2 = 0 \quad \xRightarrow{\text{θέτουμε } y=f(1)} y^2 - y - 2 = 0 \Rightarrow$$

$$\Delta = 9 \quad y_{1,2} = \frac{1 \pm 3}{2} \Rightarrow \begin{cases} y_1 = -1 \Rightarrow f(1) = -1 \text{ απορρίπτεται} \\ y_2 = 2 \Rightarrow f(1) = 2 \end{cases}$$

$$\text{Άρα } \boxed{f(1) = 2}$$

$$\beta) \quad f(x) \cdot f(y) - f(xy) = x + y \quad \xRightarrow{\text{θέτουμε } y=1} f(x) \cdot f(1) - f(x \cdot 1) = x + 1 \quad \xRightarrow{f(1)=2}$$

$$\Rightarrow 2f(x) - f(x) = x + 1 \Rightarrow \boxed{f(x) = x + 1}$$