1.11

α) $f(x)=\dfrac{2x}{x^2+1} \Rightarrow f\left(\dfrac{1}{x}\right)=\dfrac{2\cdot\frac{1}{x}}{\left(\frac{1}{x}\right)^2+1} \Rightarrow f\left(\dfrac{1}{x}\right)=\dfrac{\frac{2}{x}}{\frac{1}{x^2}+1} \Rightarrow f\left(\dfrac{1}{x}\right)=\dfrac{\frac{2}{x}}{\frac{1+x^2}{x^2}}$

$\Rightarrow f\left(\dfrac{1}{x}\right)=\dfrac{2x}{1+x^2} \overset{f(x)=\frac{2x}{1+x^2}}{\Rightarrow} f\left(\dfrac{1}{x}\right)=f(x)$

β) Έχουμε ισοδύναμα

$f(x)\le 1 \overset{f(x)=\frac{2x}{1+x^2}}{\Leftrightarrow} \dfrac{2x}{1+x^2}\le 1 \overset{1+x^2\ge 0}{\Leftrightarrow} 2x\le 1+x^2 \Leftrightarrow 0\le 1+x^2-2x \Rightarrow$

$\Rightarrow 0\le (1-x)^2$, που ισχύει